THE HEBREW UNIVERSITY OF JERUSALEM
The Leonard Davis Institute of International Relations

THE EQUILIBRIUM PRICING OF EXCHANGE RATES AND ASSETS WHEN TRADE TAKES TIME

by

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Policy Studies • 20

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INTRODUCTION

The purpose of this paper is to derive some properties of the equilibrium spot exchange rate when trade in goods takes time. We show that when trade takes time, the arbitrage properties of general equilibrium are sufficient to give rise to systematic deviations from the Law of One Price. These deviations are not temporary nor are they caused by frictions in the system, and they do not tend to disappear in the long run. As a result, even though assets are traded instantaneously, deviations from the LOP give rise to foreign exchange risk endogenously. Foreign exchange risk then causes residents of different countries to perceive the riskiness of the same financial asset differently, depending on their domicile. We use this as the definition of foreign exchange risk in this paper. We show that the risk factor attached to a foreign asset depends neither on the variability of the exchange rate nor on the pattern of the asset's cash flows in the foreign currency. This risk factor depends only on the deviations of the exchange rate from the LOP, and it enters linearly into the valuation of all foreign assets in the same way. As a result, the stability of the consumption beta of a given cash flow is neither a necessary nor a sufficient condition for the stability of the foreign exchange risk premium or of the overall risk premium attached to foreign cash flows. Finally, we show that the forward premium is a function both of the foreign exchange risk and of the relative inflation risks in the two countries.

In the remainder of this introduction, we survey briefly the literature on the relation between the LOP and the exchange rate. This literature is very extensive, and it is not possible to review it adequately in a short space. As a result, the discussion that follows is necessarily incomplete.

Most frequently, exchange rate determination models start with the assumption that prices of traded goods are equalized, adjusted for exchange rates, i.e., the LOP holds for traded goods. The typical conclusion of these models is that deviations of the exchange rate from its Purchasing Power Parity value (commonly referred to as PPP) are temporary. These temporary deviations from PPP generally depend on a variety of market imperfections, such as non-tradability of some goods or sluggish price adjustment due to nominal long-term contracts (for typical examples see Dornbusch 1973, 1976, Frenkel 1976, Johnson 1976, Stockman 1980, 1983). Roll (1979) shows that the stochastic counterpart of such models is that PPP should hold in expectations only. Aizenman (1983) shows how transactions costs affect the LOP assumption, and Cornell (1979) shows that even if the LOP holds for each good, relative price changes will result in systematic violations of PPP.

The general result on asset pricing rules for foreign assets is that if the LOP holds, then the riskiness of an asset to domestic and foreign investors is identical. In this case the International Asset Pricing Model (IAPM) that prices assets is analogous to the one-country Capital Asset Pricing Model (CAPM) but a "world market index" replaces the one-country market index. At the same time, the forward rate is shown to be a biased predictor of the future spot rate, in general, because of a risk premium associated with the relative inflation rates in the countries involved. It is shown also that PPP is sufficient for the absence of foreign exchange risk, if some restrictions are placed on the utility function (see Grauer, Litzenberger, and Stehle 1976, Fama and Farber 1979, Dornbusch 1980, and Stulz 1981, 1982. For a survey of the literature, see Adler and Dumas 1981).

If the LOP (or PPP) do not hold, foreign exchange risk may arise, depending on the nature of the deviations.[1] Furthermore, the results in the literature suggest that if foreign exchange risk exists, it is likely to be related to deviations from PPP. For discussions and models see Solnik (1974), Adler and Dumas (1975), Kouri (1976), and Frankel (1979a, 1982).

Another strand in the literature deals exclusively with the financial markets. In this literature it is shown that if there are market imperfections, the IAPM is likely to include some form of an individual "country risk" index in addition to the world "market risk" index (IAPM's are derived in Ross and Walsh 1980, Solnik 1974, Stulz 1981). This literature studies the impact of foreign exchange risk on asset pricing, under the assumption that this risk exists. The foreign exchange risk is not derived in these models.

The reliance of theoretical models on the LOP holding for traded goods at all times and PPP holding in the long run stands in sharp contrast to the conclusions of empirical investigations. The conclusion that emerges in the empirical literature is that PPP does not hold well and the LOP holds only loosely, even for well-defined commodities (see Officer 1976, Isard 1977, Kravis and Lipsey 1978, Protopapadakis and Stoll 1983, 1986, and Shapiro 1983). Exceptions are Cornell (1979) and Roll (1979). It seems highly desirable, therefore, to study the implications of a model in which the LOP (and PPP) need not hold for any good, and in which potential violations of the LOP and the related foreign exchange risk come from a well-defined and tractable structural feature of the model, rather than from a superimposed ad hoc imperfection.

One simple way to allow for potential deviations from the LOP is to postulate that exporting and importing cannot be done instantaneously. This structural feature puts trading activity with a foreign country on the same basis as investment, because in both cases economic agents have to make decisions that have uncertain outcomes in the following period.

In what follows we explore the relation between the exchange rate and the LOP in such a framework, using an Arrow-Debreu model. Section I of the paper briefly outlines the model, the essential assumptions, and the way Arrow-Debreu prices are used (the model is specified completely in Appendix I). In each state of the world there are real and nominal Arrow-Debreu prices for each future state. Simple arbitrage relations are shown to hold between the real (commodity) state prices and the nominal state price of each country, and between the nominal state prices of the two different countries.

Section II of the paper shows how the state prices can be used to price the exchange rate. First order conditions for importing and storage are shown to determine the exchange rate and its deviations from the LOP. These results are combined and summarized in Theorem 1 of the paper, in Section III. Section IV applies the results to the pricing of financial assets and derives a general pricing relation. Section V discusses the pricing of the forward exchange rate. Section VI concludes the paper.

I. REAL AND NOMINAL STATE PRICES IN AN INTERNATIONAL CONTEXT

We shall consider a two-country, two-good, world in which each country has its own currency. There are two countries A, B, and two consumption goods. Country A produces good x, and country B produces good y. Consumers in each country consume both goods.[2] Each country exports some of its output while importing some of the other country's output. Consumer preferences in each country are summarized by a utility function attributed to a "representative" consumer.

Importing takes time. Imported goods ordered at time t will arrive only at time t+1. As a result, the imported good to be consumed in the next period must be purchased from the foreign country this period and shipped. We assume that transport costs are zero. Consumers may store the imported good, in storage does not depreciate the good. Physical depreciation in storage and transport costs can be added to the model readily, but neither materially affect any of the results.

Output for the next period is obtained by investing in available technologies in this period. We assume that there exist a sufficient number of alternative production processes (with sufficient diversity in outcomes) so that uncertainty is spanned and markets are complete. We use Radner's (1972) notation to describe the sequential equilibria. The states of the world are the nodes of a tree. A state n is a combination of time, t, and an event which occurs at that time. We denote by n+ the set of all states that can follow a given state n directly (at time t+1); when convenient we call n+ the <u>successors</u> of n. Each state of the world n is the successor to precisely one previous state of the world, denoted by n-. Where no confusion ensues, we shall also use n+ to refer to any of the specific successors to n. The formal model is presented in Appendix 1 of the paper.

Consumers can purchase shares in both countries' firms. Trading in financial assets and the transfer of cash flows is costless, and they are instantaneous, in contrast to importing. Since this is a complete markets model, the two representative consumers can effect their state-by-state consumption plans by taking appropriate positions in the forward markets.

All required forward markets exist. However, except for the forward exchange rate, we do not compute any of the forward prices.

In this section we outline the asset-pricing results which follow from the model. Most of these results follow from the standard Arrow-Debreu approach to asset pricing (see Arrow 1964, Debreu 1959), with suitable adjustments made for the multi-good, multi-currency aspects of the model.

Consider a two-country world with sufficiently complete markets. At any state of the world n, consumers in each of the two countries will use state prices to price assets which give returns at states n+ that follow n. In what follows we consider only consumers in country A; symmetric statements can be made about country B-consumers.

In the simplest imaginable two-country, two-good, model, these asset returns can be of three types:

1. Assets which give real (good-denominated) returns: An asset of this type traded at state n promises to deliver one unit of good x (or good y) if a given future state n+ occurs.

2. Assets which give A-currency returns: An asset of this type traded at state n promises one unit of A-currency if a given future state n+ occurs.

3. Assets which give B-currency returns: An asset of this type traded at state n promises one unit of B-currency if a given future state n+ occurs.

In equilibrium an asset will be priced by either <u>real</u> or <u>nominal</u> state prices, depending on whether the asset promises to deliver real (i.e., good-denominated) or nominal (i.e., currency-denominated) quantities in the future. Let $q_{ax}(n+)$ denote the country A price at state n to deliver one unit of good x in state n+; similarly, $q_{ay}(n+)$ is the state n price of one unit of good y to be delivered in state n+ in country A. $q_{ax}(n+)$ and $q_{ay}(n+)$ are called the <u>real state prices</u>. The state n price, in country A, for one unit of country A-currency to be delivered in state n+ is denoted by $Q_a(n+)$ and it is called the <u>nominal state price</u>. Real and nominal state prices in country B are denoted by $q_{bx}(n+)$, $q_{by}(n+)$, and $Q_b(n+)$ respectively.

It is easily shown (see Appendix 1) that the real state price at state n for a future state n+--for a given good, in a given country--is the marginal rate of substitution in that country between the good at state n and the same good at state n+. This result is standard in the economic literature. It is also straightforward to show that the nominal state prices in each country are the real state prices adjusted by the price appreciation of the good, where $p_{ax}(n)$, $p_{ay}(n)$ are the money prices of goods x and y in country A (the result is proved in Appendix 1):

$$(1) \qquad Q_a(n+) = q_{ax}(n+)\,\frac{p_{ax}(n)}{p_{ax}(n+)} = q_{ay}(n+)\,\frac{p_{ay}(n)}{p_{ay}(n+)}.$$

Intuitively, the price quotients in equation (1) translate a nominal return to an equivalent real return (whether in the x or in the y commodity), and these real returns are priced by the relevant real state prices to produce the nominal state prices.[3]

II. SOME FUNDAMENTAL RESULTS ON EXCHANGE RATES AND TRADE

II.A. Some Preliminary Results

The objective of this section is to describe how the imported good is priced when trade takes time. But before developing this result we develop a number of preliminary pricing results.

First we note that the state prices may be used to derive the real and nominal interest rates. The country j price, in state n, to deliver one unit of good x (or y) in <u>every</u> state n+, i.e., the price of a riskless real bond in good x (or good y) is given by:

$$(2) \qquad \frac{1}{1+r_{jx}(n)} = \sum_{n+} q_{jx}(n+), \qquad \frac{1}{1+r_{jy}(n)} = \sum_{n+} q_{jy}(n+),$$

where $r_{jk}(n)$ is the country j riskless real rate of interest in terms of commodity k (k=x,y).

The country j price, in state n, to deliver one unit of country j currency in <u>every</u> state n+, i.e. the price of a riskless nominal bond (in j currency), is given by:

$$(3) \qquad \frac{1}{1+i_j(n)} = \sum_{n+} Q_j(n+),$$

where $i_j(n)$ is the country j nominal interest rate.

Prices of financial assets originating in either country are linked because assets are traded instantly. Proposition 1, below, shows the asset price arbitrage condition that must hold across the two countries. This condition is the state-by-state counterpart of the "uncovered interest arbitrage" proposition often postulated in the literature. This arbitrage condition is a key ingredient in the determination of the exchange rate.

We define the <u>exchange rate at state n</u>, <u>e(n)</u> as the number of B-dollars obtainable for one A-dollar at state n:

$$e(n)\$A(n) = \$B(n).$$

Proposition 1:

$$Q_b(n+)e(n) = Q_a(n+)e(n+) \tag{4}$$

Proof:

Consider an A-consumer who has one A-dollar at state n. She can invest this in an A-security which pays off in a specific future state n+, getting back $1/Q_a(n+)$ A-dollars in state n+. Alternatively, she can exchange her A-dollar for $1/e(n)$ B-dollars and invest in a B-security which pays off in state n+; in this case she will receive:

$$\frac{1}{e(n)} \cdot \frac{1}{Q_b(n+)} \tag{5}$$

B-dollars in state n+. Converting this back to A-dollars at the state n+ exchange rate, and setting equal the returns (in A-dollars) from the two strategies gives:

$$\frac{e(n+)}{e(n)Q_b(n+)} = \frac{1}{Q_a(n+)}, \tag{6}$$

which concludes the proof.

II.B. Trade, Exchange Rates, and the LOP

Now we turn our attention to importing. As the following proposition shows, if importing takes place at state n, then the relation between contemporaneous good prices in the two countries is unambiguously determined:

Proposition 2: In equilibrium,

$$p_{ay}(n) \leq [1+r_{ay}(n)]p_{by}(n)e(n), \tag{7}$$

with equality holding if at state n country A has ordered good y from country B (to arrive at states n+).

<u>Proof</u>:

The proof is a good example of the power of the state-preference approach. The equilibrium condition for importing is that importing either must be unprofitable in net present value (NPV) terms (in which case there will be no importing), or it must have a marginal profit of zero (again, in NPV terms). Since at state n the country A importer purchases one unit of y in country B for $e(n)p_{by}(n)$, and sells it one period later (when it arrives in country A) for $p_{ay}(n+)$, this gives the equilibrium condition:

$$(8) \qquad \sum_{n+} Q_a(n+)p_{ay}(n+) - e(n)p_{by}(n) \leq 0,$$

with equality holding if imports are ordered at state n. Substituting equation (1) for $Q_a(n+)$, rearranging, and carrying out the summation results in equation (7). Q.E.D.

Proposition 2 shows that as long as importing takes place, the LOP is <u>systematically</u> violated, except for the case where $r_{ay}(n) = 0$, i.e., a zero real interest rate in terms of the importable good y. Furthermore, the deviations of the price of the importable good from its LOP value are not constant, and they depend on the real interest rate in terms of the importable good. The intuitive explanation of Proposition 2 is that, for importing to take place, the importer has to be compensated for the cost of importing, which is the real interest rate in terms of the importable good. The importer is compensated by being able to sell the imported good at a price that is higher than in the country in which it is produced. Only if the real interest rate happens to be zero will the price of the importable good be equal in the two countries, when adjusted by the exchange rate (see Roll 1979 for a similar result).

The above result is symmetric, in the sense that it holds also for the price of the importable good in the other country:

Proposition 3: In equilibrium

$$(9) \qquad e(n)p_{bx}(n) \le p_{ax}(n)[1+r_{bx}(n)],$$

with equality holding only if country B has ordered good x from country A at state n.

Propositions 2 and 3 imply that relative prices in the two countries diverge as a function of the importable goods real interest rates. Corollary 1 formalizes this result:

Corollary 1: Suppose that at state n both countries order imports. Then the relation between the country A and the country B relative prices is given by

$$(10) \qquad \rho_b(n) = \rho_a(n)\,\frac{1}{[1+r_{ay}(n)][1+r_{bx}(n)]},$$

where

$$(11) \qquad \rho_b(n) \equiv \frac{p_{jy}(n)}{p_{jx}(n)},$$

is the relative price of good y in terms of good x in each country j=a,b. The expression is derived by multiplying together equations (7) and (9).

Propositions 2 and 3 are derived from conditions relating to the importing of the foreign good. Now we turn to the conditions under which goods may be stored. It is clear that the home good will never be stored, because the investment technology guarantees a higher rate of return. But it is possible that the home country will choose to store some of the foreign good. Proposition 4 shows the conditions under which such storage will take place. Though the foreign good may be stored for consumption in the successor states n+, it will never be stored in quantities that would allow reexporting to the foreign country. The reason is that reexporting always will be dominated in equilibrium by importing less in period t, and investing the difference in the foreign country. Since importing and then reexporting takes two periods, investing in the foreign country increases welfare. In a manner similar to the derivation of the previous propositions, we derive a first-order condition for storing:

<u>Proposition 4</u>: In equilibrium,

$$(12) \qquad 1 \geq \sum_{n+} q_{ay}(n+) = \frac{1}{1+r_{ay}(n)},$$

with equality holding if at state n country A is storing the foreign good y for consumption at successor states n+.

<u>Proof</u>:

Storage of good y costs $p_{ay}(n)$ per unit in state n and gives $p_{ay}(n+)$ per unit at each state n+. In equilibrium, the state-dependent net present value of storing cannot be positive. Therefore,

$$(13) \qquad \sum_{n+} Q_a(n+)p_{ay}(n+) - p_{ay}(n) \leq 0.$$

The result now follows from equation (1).

The intuition behind Proposition 4 is clear: We model storage as a costless activity that involves the riskless transfer of a good from one state to all of that state's successors. It is only when the real interest rate in the stored good is zero or negative that storage can be profitable; furthermore, in equilibrium, the real return from storage (zero) will prevent the real interest rate in the stored good from becoming negative.

III. PROPERTIES OF THE SPOT EXCHANGE RATE

The first-order conditions for storage and importing shown above allow us to derive the following taxonomy for the value of the exchange rate relative to its LOP value:

Theorem 1:[4] The properties of the spot exchange rate, storage, and importing, are related as follows:

Foreign good ordered in state n for delivery at n+?	Foreign good stored at state n for possible consumption at n+?	Exchange rate e(n) vs. its LOP value?
yes	yes	e(n) = LOP
yes	no	e(n) < LOP
no	yes	e(n) > LOP
no	no	impossible case under the assumptions of the model

Proof:

The proof of the Theorem follows directly from Propositions 2-4,

Q.E.D.

Theorem 1 shows that the LOP will hold for a good only if the good is imported and stored simultaneously. We show below that simultaneous storage and importing cannot occur frequently in general equilibrium. This means that the LOP will hold only infrequently and that deviations from the

LOP are systematic (i.e. the result of consumer maximization in general equilibrium) and not due to transitory frictions in the system. Furthermore, there are no general equilibrium forces which will cause the deviations from the LOP to disappear in the long run.

The argument that the LOP holds infrequently at best is as follows. It is clear from Proposition 4 that the LOP holds for good y only when $r_{ay}(n) = 0$ (or for good x when $r_{bx}(n) = 0$). We know also that these conditions will be fulfilled only if the expected capital gains from storing the importable good are large enough to offset the opportunity cost of storing (i.e. the interest rate in the good). But such expected capital gains will not be available regularly. If such capital gains were available regularly, it would imply that the relative price of the importable good would grow continuously. Since the importable good is produced in the foreign country every period, such continuous expected relative price appreciation is not consistent with equilibrium. Storage will tend to occur when last period's foreign output was unusually large (and therefore shipments were large) or when this period's home output is unusually small (and therefore consumption of the importable is small).

This analysis makes it clear that there is not any long-run tendency for the LOP to hold. In addition, the deviations from the LOP are a real phenomenon, and they are not influenced by monetary factors directly. The behavior of the exchange rate around its LOP value will not exhibit smooth patterns, generally. It has been shown already that if the LOP holds in a given period, it will <u>not</u> hold in at least one state in the next period, since whether or not LOP holds, storage will not take place in every state next period. Therefore, there will never be a situation in which the LOP is expected to hold with certainty. The deviations from the LOP generally are not stationary, because they depend on the real interest rates, whose distribution would be stationary only under certain restrictive assumptions.

Furthermore, because of the implied relative price shifts within each country, the probability that the LOP will hold next period is inversely related to the number of consecutive periods it has been holding. Also, the relative price relation, Corollary 1, implies that the probability that the LOP will hold is higher the lower the relative price of the importable, in any state n. This is true because the lower the relative price of the importable is today, the more likely it is that the expected capital gain will be enough to induce storage.

It is useful to discuss here the effect on our results of introducing depreciation and transport costs. If the foreign good is imported, both transport and depreciation costs, τ and δ, will be incurred. When these costs are added, the right hand side of the inequalities in Propositions 2 and 3 are multiplied by $(1+\tau)(1+\delta)$. If τ and δ are fixed, their introduction has no effect on any of the results; they only serve to make the deviations from the LOP larger. If they are state-dependent, they may add some risk premia to our pricing equations, depending on what one assumes about their covariance with the state prices. Similarly, the left hand side of the inequality in Proposition 4 is multiplied by $(1+\delta)$. This means that the minimum value of the real interest rate in terms of the foreign good becomes approximately the negative of the depreciation rate.

In the monetary, or asset, approach to exchange rate determination it is often stated that "... the exchange rate is determined in the asset markets" (see Frenkel 1976). It is interesting to note that only if there is no importing (line 3 of the table of Theorem 1) is there strict justification for this claim. The justification for claiming that the asset markets determine the exchange rate, in this case, is that the financial markets' first-order condition is the only condition that governs the exchange rate, given future expectations. As long as there is importing of the good, the goods market conditions will determine the exchange rate (equations 7 and 9) jointly with the financial markets condition.

IV. THE INTERNATIONAL PRICING OF ASSETS

One of the important issues in international finance is the pricing of assets internationally, and the impact of "foreign exchange risk" on the pricing of assets. It has been shown in the literature that if PPP holds, then the denomination of the cash flows of the asset to be priced is of no consequence.[5] Agents, regardless of their country of residence, agree on the riskiness of each asset, and they use the same risk premium to discount cash flows. A closely related issue is the pricing of real riskless bonds in various countries, and whether "real" interest rates are equalized across countries. Again the conclusion in the literature is that real rates are equalized if PPP holds.

We start by investigating the relation between real state prices in the two countries, in terms of the same good. If the real state prices are equal across countries, consumers in the two countries price cash flows identically. In particular, consumers in the two countries agree on the riskiness of any given cash flow. In such a world there is <u>no</u> foreign exchange risk, by definition, because the perceived riskiness of <u>any</u> cash flow does not depend on the domicile of the consumer. The fact that there are two countries and two currencies is of no importance for the evaluation of risk. Note that there is no foreign exchange risk in this instance, despite the variability of the exchange rate, because consumers receive the same real flows from an asset regardless of their country of residence. But if real state prices are not equal in the two countries, there may be foreign exchange risk, because the real flows may depend on the country of residence.

The next proposition shows that real prices are not equalized if the exchange rate deviates from its LOP value. First we need the following definitions: Let

(14) $\quad ep_x(n) \equiv p_{ax}(n)/p_{bx}(n)$ and $ep_y \equiv p_{ay}(n)/p_{by}(n)$,

denote the LOP value of the exchange rate at state n. Denote deviations from the LOP value by

(15) $\quad \Delta_k(n) = e(n)/ep_k(n)$, $k = x, y$.

Proposition 5: Real state prices at any state n in terms of either good are not equalized unless:

(i) the LOP holds now and will hold in every state in the next period, or,

(ii) deviations from the LOP will remain constant from state n to every successor state n+.

Proof:

Recall from Proposition 1 that $e(n)Q_b(n+) = Q_a(n+)e(n+)$. Substitute the first-order condition (1) into this equation to derive:

$$(16) \qquad q_{ay}(n+) = q_{by}(n+) \frac{\Delta_y(n)}{\Delta_y(n+)},$$

$$(17) \qquad q_{ax}(n+) = q_{bx}(n+) \frac{\Delta_x(n)}{\Delta_x(n+)},$$

which proves the proposition.

It follows from Proposition 5 that since real state prices need not be equal in the two countries, in general, real interest rates will not be equal across countries.

Now we are in a position to show that deviations from the LOP create foreign exchange risk. We show in Theorem 2, below, that a consumer's perception of the overall riskiness of an asset depends on the intrinsic risk of the asset's cash flows, on the consumer's country of residence, and on the currency in which the asset's cash flows are denominated. Foreign exchange risk arises because deviations from the LOP cause real state prices, and hence real interest rates, to differ across the two countries. Theorem 2 shows that the breakdown of the LOP implies that agents from different countries use different risk premia to discount a given cash flow. Also it shows that the resulting foreign exchange risk premium is proportional to the real interest rate term structure premium.

Theorem 2: Consider an asset in country B that has nominal B-dollar cash flows of CFB(n+) in each state n+. Then the value of the asset in B-dollars to a country B-consumer in state n is given by:

$$(18) \qquad VB(n) = p_{by}(n)\{\frac{E\ cfb(n+)}{1+r_{by}(n)} + cov[q_{by}(n+), cfb(n+)]\},$$

where $cfb(n+) = CFB(n+)/p_{by}(n+)$, the real (y-good) cash flows.

At the same time, the value of the <u>same</u> asset in B-dollars to a country A-consumer in state n is given by:

$$\text{(19)} \qquad VA(n) = p_{by}(n)\{\frac{E\ cfb(n+)}{1+r_{ay}(n)}\ \frac{E\ \Delta_y(n+)}{\Delta_y(n)} + cov[q_{by}(n+),cfb(n+)]$$

$$+ \frac{E\ cfb(n+)}{\Delta_y(n)}\ cov[q_{ay}(n+),\Delta_y(n+)]\},$$

where $\Delta(n) = e(n)/ep_y(n)$, the deviations of the exchange rate from it's LOP value, and where $E(\cdot)$ is the expectation conditional on state n. The A-dollar price of the asset is $e(n)VB(n) = e(n)VA(n)$.

<u>Proof</u>: Theorem 2 is proved in Appendix 2 to the paper.

<u>Remarks</u>:

1. Theorem 2 ties together various strands of the literature, and it shows the importance of the LOP in pricing assets internationally and assessing their riskiness. Equations (18) and (19) show that if the LOP holds, then the riskiness of a particular cash flow is the same for both country A and country B-agents, and the risk premia attached to that cash flow by both agents are identical. This is because the second covariance term in country A-agents' pricing equation (equation 19) is identically zero.[6] To see this recall that if the LOP holds, $\Delta(n+)=\Delta(n)=1.0$ by definition, and $r_{ay}(n) = r_{by}(n)$ -- real interest rates are equalized. If the LOP does not hold, however, the risk premia depend on the agents' domicile.

2. The first covariance term in each of the two pricing formulas in Theorem 2 is identical. That term is the intrinsic systematic risk of the cash flows, or the consumption beta of the cash flows.[7] The consumption beta in both expressions is evaluated with respect to <u>country B</u> real state

prices. In particular, all consumers use the state prices of the country in whose currency the cash flows are denominated. In contrast to the existing literature, the asset's systematic risk is not measured with respect to a "world market portfolio." The counterpart of the world market portfolio beta is that agents from both countries measure the intrinsic risk of the asset with respect to <u>the same</u> real state prices. This is not a trivial result, because different agents face different real state prices in their own countries.

Country A-agents also perceive risk that arises because the currency of denomination of the assets is different. This risk term is due to <u>foreign exchange risk</u>. The foreign exchange risk in this model is the systematic risk (the beta) of the deviations from the LOP and it is evaluated with respect to country A real state prices.[8] Though both foreign and domestic country agents perceive the same intrinsic riskiness in the cash flows, foreign agents perceive an additional risk that is created because the LOP generally does not hold, and they price that risk with the local state prices. Our result is in accord with the conclusion in some of the literature, that foreign exchange risk is generated as a result of deviations from PPP. This risk is not connected in any direct way to the variability of the exchange rate. Similar results have been derived in the literature by Solnik (1974), Stulz (1981) and others, by assuming an exogenous foreign exchange risk. The uniqueness of this model is that deviations from the LOP and the resulting foreign exchange risk are endogenous.

3. It follows from equation (7) above that deviations of the exchange rate from the LOP are general, and they are related to the real interest rate in the traded good. In general,

$$\Delta(n+) = \frac{1}{1+r_{ay}(n+)}, \tag{20}$$

as long as importing takes place. The interest rate $r_{ay}(n+)$ is the one-period interest rate that will be known in period t+1 (i.e., it refers to states n++ conditional on a specific state n+). The intuition of this result is that when trade takes time, the importer must be recompensed at the real rate of interest in terms of the imported good. But since at time t the time t+1 interest is unknown, the deviations from the LOP are stochastic. To the extent that they covary with today's state prices, these deviations give rise to a risk premium in the pricing of foreign cash flows.

4. The foreign exchange risk, then, is the covariance of the distribution of state prices available today, $q_{ay}(n+)$, with the distribution of short-term interest rates. The foreign exchange risk premium is proportional to the real interest rate term structure premium.[9]

5. A unique feature of the pricing equations of Theorem 2 is that the foreign exchange risk premium is unrelated to the characteristics of the underlying cash flows. Thus the <u>same</u> risk premium applies to <u>all</u> foreign assets. As a result, stability of the consumption beta of a given cash flow is neither a necessary nor a sufficient condition for the stability of the foreign exchange risk premium and of the overall risk premium attached to foreign cash flows.[10]

6. The risk-pricing formula derived here is similar to the IAPM derived by Solnik (1974) in a partial equilibrium framework. Solnik (1974) shows that if there is foreign exchange risk, consumer portfolios will be made up of the risk-free asset, the world market portfolio hedged for

foreign exchange risk, and a foreign exchange risk fund (see also Kouri 1976, Ross and Walsh 1980). An alternative IAPM has been used frequently in the empirical literature. A common formulation in this literature asserts that the risk premium that arises from the "foreignness" of the asset can be measured by the covariance of the asset cash flows with the national market portfolio, after adjusting for the "world portfolio" (see Lessard 1976, Stehle 1977). Thus, in this version of the IAPM, both risk measures are assumed to be asset specific. In our model, the foreign exchange risk is not asset specific; it only relates to the riskiness of the exchange rate. Furthermore, the foreign exchange risk is measured relative to the foreign country real state prices, while the cash flow risk is measured relative to the home real state prices.

V. PRICING THE FORWARD RATE

The pricing relations developed in the previous section can be used to derive an expression for the forward exchange rate, and to determine the impact of various risks on that rate. It is shown here that the forward rate is not equal to the expected spot rate, both because of the systematic risk associated with the relative inflation rates in the two countries, and because of the existence of foreign exchange risk. We single out the forward exchange rate here, because of the extensive theoretical and empirical investigations of the relation between the forward and expected future spot rates. However, similar relations can be derived for any forward price of interest.

Since asset prices are arbitraged internationally, the forward rate must be such that the net present value (NPV) to the country A-consumer of covering a B-currency position forward is zero. This relation leads to:

$$(21) \qquad \frac{1}{e(n)} \sum_{n+} Q_a(n+)[e(n+) - f(n)] = 0,$$

where f(n) is the one-period forward rate in state n. Manipulating equation (21) yields,

$$(22) \qquad f(n) = Ee(n+) + [1+i_a(n)]\text{cov}[Q_a(n+),e(n+)].$$

Equation (22) makes clear that the forward rate is a biased predictor of the expected spot rate, generally, and that the bias is related to the systematic risk of the exchange rate fluctuations. Though equation (22) is a complete characterization of the forward rate, it conceals important determinants of the forward risk premium, because it is not possible to tell how the forward rate is affected by whether or not the LOP holds.

To get more insight into the makeup of the risk premium, rewrite the covariance term in equation (22) by using equation (1):

$$(23) \qquad \text{Cov}[Q_a(n+),e(n+)] = \text{cov}[Q_a(n),\Delta_y(n+)ep_y(n+)],$$

$$= ep_y(n)\text{cov}[Q_a(n+),\Delta_y(n+) \frac{\pi_{ay}(n+)}{\pi_{by}(n+)}],$$

where $\pi_{jy}(n+) = p_{jy}(n+)/p_{jy}(n)$, the inflation rate in each country (j=a,b) in terms of good y. Equation (22) thus becomes,

$$(24) \qquad f(n) = Ee(n+) + ep_y(n)[1+i_a(n)]\text{cov}[Q_a(n+),\ \Delta_y(n+) \frac{\pi_{ay}(n+)}{\pi_{by}(n+)}].$$

The difference between the forward rate and the expected future spot rate (often referred to as the forward premium) depends both on the risk of relative inflation and on foreign exchange risk. Equation (24) shows that one risk factor is related to the systematic part of the variability of relative inflation rates in the two countries, measured in terms of good y. This is similar to the results in Grauer, Litzenberger, and Stehle (1976), Fama and Farber (1979), and Stulz (1982). The other risk factor that makes up the risk premium is the systematic part of the deviations from the LOP, i.e., the foreign exchange risk. The existence of foreign exchange risk modifies the risk premium that is due to relative inflation rates. Even if there is no relative inflation risk, the forward rate will not equal the expected future spot rate, in general, because deviations from the LOP will exist, for at least one state, in the next period.

VI. CONCLUSION

In this paper we analyze the properties of the spot and forward exchange rates and the appropriate asset pricing rules when trade in goods is not instantaneous. The fact that trade takes time causes systematic deviations of the exchange rate from its Law of One Price (LOP) value. These deviations depend on the real interest rate in terms of the importable good. The spot rate may occasionally attain its LOP value, but it has no systematic tendency to reach the LOP value in the long run, or to remain at the LOP value, if it reaches it. Furthermore, the deviations from the LOP need not be stationary.

Deviations from the LOP mean that the riskiness of assets is different to agents in different countries. Consequently, agents in different countries price the <u>same</u> cash flows by using different risk premia. In particular, agents in both countries agree on the measure of intrinsic riskiness of the cash flows (the consumption beta), but agents in the "foreign" country price the asset by adding a risk premium related to the foreign exchange risk.

The foreign exchange risk is not related to the systematic (or unsystematic) variability of the exchange rate. Rather it is the systematic variability of the deviations of the exchange rate from its LOP value that gives rise to foreign exchange risk. Also we show that the forward exchange rate generally is not equal to the future spot rate. The forward premium depends both on the foreign exchange risk and on the riskiness of relative inflation in the two countries.

APPENDIX 1

A Two-Country Model in Which Trade Takes Time

In this appendix we specify the complete model from which we derive the state pricing results of Section 2.

1.1. Features of the Model:

We specify a world with two countries. Consumer preferences in each country are summarized by a utility function attributed to a "representative" consumer. Each country produces one good, and it exports some of its output while importing some of the other country's output. Consumers can purchase shares in both countries' firms. Output for the next period is obtained by investing in available technologies in this period. The imported good to be consumed in the next period must be purchased from the foreign country this period and shipped. For simplicity we assume that when a good is stored or shipped, it does not depreciate. In contrast to importing, asset trade is instantaneous and costless. Future outcomes of production and prices are uncertain. We consider a model with a tree-like time-state structure, in which both the outcomes for each state and the probability of any state occurring are known in advance.

As stated in Section II, we use Radner's (1972) notation to describe uncertainty. A state n is a combination of time, t, and an event which occurs at that time. We denote the set of all states that can follow state n directly by n+ (at period t+1). Each state of the world n is the successor to precisely one previous state of the world, denoted by n-. We assume that all economic agents agree on the probability distribution of the states.

Production: There are two countries A and B, and two consumption goods x and y. Country A will produce good x only, and country B produces good y only. Only good x can be used to produce more x and only good y can be used to produce more y. We assume that there exist a sufficient number of alternative production processes (with sufficient diversity of outcomes) so that uncertainty is spanned, and markets are complete. Each production process delivers output in each state in period t+1.

Consider the representative consumer in country A. Suppose this consumer has to make production input (investment) choices at state n, and suppose that there are N+ possible successors to state n; i.e., n+ =1,...,N+, and there are I independent production processes available ($I \geq N+$). Then at state n the consumer invests in I good-x inputs, $z_x^1(1),\ldots,z_x^I(N+)$, each assigned to its specific production process. The i'th production process, denoted by $g_x^i[z_x(n+);n+]$, delivers $g_x^i[z_x^i(n+);1]\ldots,g_x^i(n+);N+]$ in each of the n+ states that succeed n. Similarly, the representative consumer in country B choose inputs of good y, $z_y^i(n+)$, at state n, which are used to produce at the successor states to n. Where no confusion is caused, we write $g_k^i[z_k^i(n+);n+]$, k=x,y, both for specific production in any successor to n and as a generic expression for production in the set of successor states to n. If n is a time-state occurring at time t, planned production for states n+ (the successors of n) is given by:

(1.1.1) production of x in state n+ $= \Sigma_i g_x^i[z_x^i(n+);n+]$,

(1.1.2) production of y in state n+ $= \Sigma_i g_y^i[z_y^i(n+);n+]$; for all n+.

Total investment in the production of x and y at n is given by $z_x(n+)$, $z_y(n+)$, respectively. We assume that the production functions are concave and differentiable and that the usual Inada conditions hold:

$$(1.1.3) \quad g'_{ik}[z^i_k(n);n] \equiv \frac{d\ g^i_k[z^i_k(n);n]}{d\ z^i_k(n)} > 1, \text{ for all } n, \text{ and } g'_{ik}(0;n+) \to \infty,$$

$i=1,\ldots,I;\quad k=x,y.$

The existence of a sufficient number of production functions ensures that the amount of consumption in each state is independent of consumption in the other states, and that markets are complete in the usual sense (Benninga and Protopapadakis 1986 discusses the relation between production functions and market completeness).

Consumption: There is a representative consumer in each country, with a generalized utility function:

$$(1.1.4) \quad U_j[x_j,y_j,M_j;\ H_j], \qquad j=a,b,$$

where x_j, y_j are consumption vectors of goods x, y, for all state n, M_j is the vector of nominal money balances (home currency) for all n, and H_j is a vector of nominal prices (or a price index) that deflates the nominal money holdings. Prices enter the utility function because, as observed by Samuelson (1968), this is necessary when money is in the utility function (for more detailed discussion of the issues involved, see Benninga and Protopapadakis 1984 and Leroy 1984).[11] Consumers are restricted to hold home currency only. This is not a crucial assumption for our purposes, because we do not intend to deal with issues related to currency substitution (for models that allow currency substitution, see Stockman 1980, Kareken and Wallace 1981). Including foreign money as a separate argument in the utility function does not alter any of our results, because, as long as both moneys are held in equilibrium, the trade and storage first-order conditions we exploit will continue to be satisfied. If the two moneys are assumed to be perfect substitutes, then one of them may not be held by anyone. But even in this case, the first-order conditions we use will still hold, and deviations from the LOP will continue to exist. We assume that the Inada conditions hold (marginal utility at zero consumption of any good, and money, is infinite), so that there will be positive consumption and positive money holdings in all states.

Foreign Trade and Storage: It takes one time period to ship goods from one country to another. Country A-consumers purchase $s_y(n)$ amount of good y from country to another. Country A-consumers purchase $s_y(n)$ amount of good y from country B in state n (period t) and ship it to arrive at states n+ (period t+1). Similarly, country B-consumers purchase $s_x(n)$ amount of good x from country A in state n, and they ship it to arrive at states n+. Transport costs are zero. Note that it is not possible to make state-contingent shipments, so that the quantity of the imported good available for consumption is the same for all states n+ (in period t+1).[12]

It is possible to store goods. It is obvious that there will be no storage of x in country A, or of y in country B, as long as the marginal product of capital is positive. However, it is conceivable that there will be storage of the imported good. We designate the quantity of the imported goods stored in state n by $w_y(n)$ (y stored in country A) and $w_x(n)$ (x stored in country B). Physical depreciation is zero. Since there is no explicit or implicit utility derived from it, storage is purely a speculative activity, and whether storage takes place depends only on the expected capital gains.

Asset Markets: Assets can be traded instantaneously and costlessly across countries, and consumers are free to purchase domestic and foreign securities. In state n, consumers in each country j (j=a,b) own securities that represent $\theta_{jk}(n)$ proportion of the value of the output of the k'th good (k=x,y). In state n, each consumer receives the proceeds of the output, and he purchases $\theta_{jk}(n+)$ shares of the value of the future output of each good. Thus the country A consumer purchases $\theta_{ax}(n+)$ shares of the value of the future x-output and $\theta_{ay}(n+)$ shares of the value of the future y-output; θ_{jk} can be either positive or negative (i.e., there are no short sale restrictions in asset markets).

Each consumer also holds the currency of his home country. He enters the period holding $M_j(n-)$ money (j=a,b), and at the end of the period he chooses to hold $M_j(n)$ money.

1.2. The Maximization Problem:

The model outlined above results in the following maximization problem: country A-consumer chooses $\{k_a(n),\ z_x(n+),\ s_y(n),\ w_y(n),\ \theta_{ak}(n+),\ M_a(n);$ for all n and k=x,y$\}$, so as to maximize a utility function:

$$\text{Max } U_a[x_a,\ y_a,\ M_a;\ H_a],$$

subject to the budget constraints (one for each n):

(1.2.1)

$$\Sigma_i\theta^i_{ax}(n)p_{ax}(n)g^i_x[z^i_x(n);n] + \Sigma_i e(n)\theta^i_{ay}(n)p_{by}(n)g^i_y[z^i_y(n);n]$$

shares in the value of the production of goods x and y, that were purchased in the last period;

$$+\ p_{ay}(n)w_y(n-) \quad + \quad p_{ay}(n)s_y(n-) \quad + \quad M_a(n-) =$$

good y stored from the previous period; imports purchased in the last period in the foreign country (which arrive this period); money balances from last period

$$\Sigma_i \theta^i_{ax}(n+)p_{ax}(n)z^i_x(n+) + \Sigma_i e(n)\theta^i_{ay}(n+)p_{by}(n)z^i_y(n+) \quad + \quad M_a(n)$$

shares in the future value of the outputs of goods x and y; new money balances;

$$+\ p_{ax}(n)x_a(n) + p_{ay}(n)y_a(n) \quad + \quad e(n)p_{by}(n)s_y(n) \quad + \quad p_{ay}(n)w_y(n),$$

consumption of good x and y; importable good shipped to arrive next period; storage of the importable good;

and subject to:

$$z^i_x(n+),\ M_a(n),\ s_y(n),\ w_y(n),\ k_a(n) \geq 0.$$

A similar problem is formulated for the representative consumer in country B.

The first-order conditions for these problems give the real and nominal state prices of Section II of the paper.

APPENDIX 2

Proof of Theorem 2

Recall that

$$VA(n) = \frac{1}{e(n)} \sum_{n+} Q_a(n+)CFB(n+)e(n+),$$

(2.1)
$$= \frac{1}{e(n)} \sum_{n+} q_{ay}(n+) \frac{p_{ay}(n)}{p_{ay}(n+)} CFB(n+)e(n+),$$

$$= \frac{1}{e(n)} \sum_{n+} q_{ay}(n+) \frac{p_{ay}(n)}{p_{ay}(n+)} \frac{CFB(n+)}{p_{by}(n+)} p_{by}(n+)e(n+).$$

Let $cfb(n+) \equiv CFB(n+)/p_{by}(n+)$. Furthermore recall $ep_y(n+) \equiv \dfrac{p_{ay}(n+)}{p_{by}(n+)}$.

Thus we may write,

(2.2)
$$VA(n) = \frac{1}{e(n)} \sum_{n+} q_{ay}(n+)p_{by}(n)ep_y(n)cfb(n+) \frac{e(n+)}{ep_y(n+)},$$

(2.3)
$$VA(n) = \frac{p_{by}(n)}{\Delta_y(n)} \sum_{n+} q_{ay}(n+)cfb(n+)\Delta_y(n+),$$

where we let let $\Delta_y(n) = \dfrac{e(n)}{ep_y(n)}$.

Now use the rule, $E(xyz) = E(x)E(y)E(z) + cov(xy,z) + E(z)cov(x,y)$. This gives:

(2.4)
$$VA(n) = \frac{p_{by}(n)}{\Delta_y(n)} \{\frac{1}{1+r_{ay}(n)} Ecfb(n+)e\Delta_y(n+) +$$

$$cov[q_{ay}(n+)\Delta(n+),cfb(n+)] + Ecfb(n+)cov[q_{ay}(n+),\Delta_y(n+)]\}.$$

Now recall that,

$$q_{ay}(n+)\Delta_y(n+) = q_{ay}(n+)e(n+)\,\frac{p_{by}(n+)}{p_{ay}(n+)}, \tag{2.5}$$

$$= q_{ay}(n+)e(n+)\,\frac{p_{by}(n+)}{p_{ay}(n+)}\,\frac{p_{ay}(n)}{p_{ay}(n)},$$

$$= Q_a(n+)e(n+)\,\frac{p_{by}(n+)}{p_{ay}(n)}.$$

Since $Q_a(n+)e(n+) = Q_b(n+)e(n)$, this last expression may be written as,

$$Q_a(n+)e(n+)\,\frac{p_{by}(n+)}{p_{ay}(n)} = Q_b(n+)e(n)\,\frac{p_{by}(n+)}{p_{ay}(n)}, \tag{2.6}$$

$$= q_{by}(n+)\,\frac{p_{by}(n)}{p_{by}(n+)}\,e(n)\,\frac{p_{by}(n+)}{p_{ay}(n)} = q_{by}(n+)\,\frac{e(n)}{ep_y(n)} = q_{by}(n+)\Delta_y(n).$$

Thus we may write:

$$VA(n) = \frac{p_{by}(n)}{\Delta_y(n)}\,\Big\{\frac{Ecfb(n+)}{1+r_{ay}(n)}\,E\Delta_y(n+) + \Delta_y(n)\operatorname{cov}[q_{by}(n+), cfb(n+)] \tag{2.7}$$

$$+ Ecfb(n+)\operatorname{cov}[q_{ay}(n+), \Delta_y(n+)]\Big\},$$

$$(2.8) \qquad VA(n) = p_{by}(n)\{\frac{Ecfb(n+)}{1+r_{ay}(n)} \frac{E\Delta_y(n+)}{\Delta_y(n)} + cov[q_{by}(n+), cfb(n+)]$$

$$+ \frac{Ecfb(n+)}{\Delta_y(n)} cov[q_{ay}(n+), \ \Delta_y(n+)]\}.$$

which is equation (19) of the text.

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FOOTNOTES

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[1]Works in this literature focus either on the LOP or PPP, depending on the framework. However, whereas the definition of the LOP is clear (prices of the same good are equal everywhere, adjusted by exchange rates), the definition of PPP is ambiguous for a multi-good world. For clarity of exposition we use only the concept of the LOP in our analysis, and we refer to PPP only when discussing results in the literature.

[2]The reason for studying a model in which all consumers consume <u>both</u> goods, is that, unlike models in which consumption bundles differ, this specification makes it possible to explore price differentials for the <u>same</u> good across countries, and it allows us to study deviations from the LOP and their consequences.

[3]It may seem reasonable to use one of the goods as a numeraire and to eliminate money prices. However, we feel strongly that it is more natural and easier (even at the expense of some additional notation) to think of the exchange rate as a nominal quantity, rather than to think in terms of relative prices of the two goods in <u>each</u> country, as well as relative prices of each of the goods <u>across</u> the two countries.

[4]Note that nowhere in the paper is there reference to first-order conditions associated with money holdings, and no discussion of monetary policy, though real money balances appear in the utility function, and we are studying aspects of a monetary equilibrium. Money supply and monetary

policy interact with real equilibrium to determine the levels as well as the paths of all the variables. However, since all the first-order conditions we study in the paper must hold regardless of monetary policy, the properties of the deviations of the exchange rate from its LOP value are not affected by monetary policy considerations. The departures from LOP are a real phenomenon, and monetary policy can have an impact on the magnitude of the deviations but only through non-neutralities that affect real interest rates and relative prices. For some representative analyses of these issues, see Johnson (1976), Frankel (1979b), Stockman (1980), Liviatan (1981), Helpman (1981), Helpman and Razin (1982) and Aizenman (1983, 1984).

[5]See Kouri (1976), Aliber (1978), Fama and Farber (1979), and Lucas (1982). Grauer, Litzenberger and Stehle (1976) show that certain trade restrictions they consider do not give rise to foreign exchange risk.

[6]This is a result well-known in the literature. See Grauer, Litzenberger and Stehle (1976), Fama and Farber (1979), and Stulz (1981).

[7]See Breeden (1979) for a derivation of the consumption beta and a discussion of its connection with the usual CAPM representation.

[8]Clearly, there are other, equivalent ways to express V(n). This particular formulation lends itself most directly to inter-country comparisons, and to an analysis of the effects of the LOP holding.

[9]See Benninga and Protopapadakis (1983). They give the following term structure relation: ${}_0R_2/{}_0R_1 = E({}_1R_2) + (1/{}_0R_1)\mathrm{cov}[q_1/\pi_1, {}_1R_2]$, where ${}_0R_1$, ${}_0R_2$ are the prices of the one-period and two-period real riskless bonds. $E({}_1R_2)$ is the expected price of the one-period bond next period, and q_1/π_1 are the probability-adjusted real state prices today.

[10]When importing is taking place, the foreign exchange risk premium is similar to the real term-structure premium, but the deviations from the LOP are adjusted by the ratio of the real state prices in the two countries. It may seem reasonable to apply the result in Benninga and Protopapadakis (1986) in this case. The result is that with concave production functions, the real term structure premium is positive if the probability distribution is symmetric and independent. However this result does not apply directly here, because the importable good is produced only in the foreign country.

[11]See Samuelson (1968) for a lucid explanation of the reasons for entering prices in the utility function if nominal money is entered in the utility function. Entering money in the utility function should be viewed as a way of summarizing the economic reality that money performs valuable services, without making it necessary to specify exactly what these services are. For detailed discussions of the issues that arise, see Lucas (1980), Benninga and Protopapadakis (1984), and Leroy (1984).

[12]We do not allow for state-contingent imports in this model both because it complicates the analysis without affecting the material conclusions, and because we could not think of a real-world situation that corresponds to state-contingent shipping (the assumption in the model is that goods spend one period <u>in transit</u>).

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